Financial and Economic Review, Vol. 16 Issue 4., December 2017, pp. 36–57.

Possible Paths for GDP Per Capita – Simulation with a Demographic Growth Model*

Éva Berde – Izabella Kuncz

To begin our paper, we point out the importance of demographic growth models by highlighting the conceptual framework of overlapping generations. We define the formulas in our own model based on Lee – Mason (2010), modifying the original framework in several respects. We present the exogenous fertility and survival rates in different demographic scenarios, and then we derive the simulation paths for GDP per capita from these. A word of caution regarding our results: the simple structure of our model disregards several factors potentially influencing growth. When concentrating solely on the impact of changes in fertility and mortality rates, our simulation results suggest that a drop in the total fertility rate, even to slightly below the replacement level, and even in the context of a relatively high survival rate, may increase GDP per capita. However, in the case of an extremely low total fertility rate and a high survival rate, an economic downturn can be expected.

Journal of Economic Literature (JEL) codes: E23, J11, O40

Keywords: overlapping generations model, total fertility rate, survival rate, simulation path, GDP per capita

1. Introduction

The exploration of the regularities in economic growth and the projection of potential future paths has excited economists since the emergence of economic thinking.[1] Nevertheless, the significance of population numbers and structures

---

* The views expressed in this paper are those of the author(s) and do not necessarily reflect the offical view of the Magyar Nemzeti Bank.

Éva Berde is a professor at the Corvinus University of Budapest. E-mail: eva.berde@uni-corvinus.hu
Izabella Kuncz is an assistant lecturer at the Corvinus University of Budapest.
E-mail: izabella.kuncz@uni-corvinus.hu

The study was financed by the Pallas Athena Domus Animae Foundation. The views expressed in this article are solely those of the authors.

The Hungarian manuscript was received on 26 June 2017.

DOI: http://doi.org/10.25201/FER.16.4.3657

[1] The first such efforts that can be cited appeared with the representatives of the classical school of economics. *Smith* (*1776*) not only presented the benefits of the division of labour, but also mentioned that the productivity growth that can be achieved through the division of labour increases economic prosperity, and facilitates the future development of the economy.

only started to be taken into account in growth models much later.[2] From the second half of the 1980s, as humanity entered the fourth stage of demographic transition, demographics determined economic developments to such an extent that demographic variables could no longer be left out of growth models.

In the fourth stage of demographic transition,[3] the mortality rate steadily declines in almost all developed countries of the world, and in parallel with this, the total fertility rate (TFR) is even lower than earlier. The total fertility rate has dropped to below the replacement level several times, which portends serious problems for the future. Fulfilling the needs of the ageing population, and, more generally, maintaining the operation of the economy may prove to be highly difficult under such circumstances. While the total fertility rate of more developed regions drops, huge net population growth characterises certain less developed areas, due to a diminishing mortality rate combined with a high fertility rate, which, however, is still lower than earlier. In these countries, the future decrease in the total fertility rate and the further drop in the mortality rate are expected, and therefore it is likely that the share of the working-age population will soar temporarily. These trends may provide an economic stimulus to the countries concerned, and the beneficial effect of the so-called first demographic dividend[4] can take hold (see, for example, *Bloom et al. 2003, Mason 2005*). Ultimately, even the ranking in the economic clout of the countries may change.

Out of the growth models, the changing demographic trends can be best captured by the so-called overlapping generations (OLG) model. In the study by *Diamond* (*1965*), which laid the foundations of OLG models, changes in the birth rate were still exogenous, however, it already differentiated between two age groups of the population. This framework was expanded by *Auerbach – Kotlikoff* (*1987*) to include further cohorts, and then the modelling tools were enhanced by *Barro – Becker* (*1989*). The article by Barro–Becker also treats the birth rate endogenously: therefore, the total fertility rate is quantified as a result of the processes in the model, and it has an impact on the development of the model's variables.

Our paper draws conclusions with respect to the potential paths of economic growth using one of the overlapping generations models. The basic model we used, *Lee – Mason* (*2010*), uses a special approach as compared to the usual OLG models. Still, it is based on Diamond's idea, i.e. the overlapping generations, and

[2] *Malthus* (*1798*) concentrated specifically on the negative impact of excessive population growth, but did not link this issue to the potential growth in productivity. Demographic factors were first taken into account in growth models by *Solow* (*1956*). The historical sequence of the demographic approaches to growth models is summarised in *Table A1* in the Appendix.

[3] For more on the demographic transition, see, for example, *Van de Kaa* (*2010*), *Frejka* (*2016*).

[4] The first demographic dividend shows the difference between the growth rate of the weighted number of workers and the growth rate of a similarly weighted number of consumers. If the number of the so-called effective workers increases more (or falls less) than the number of effective consumers, that contributes to economic growth.

the neoclassical growth model framework, in which economic growth is simulated as a function of the total fertility rate and the mortality rate. One simplification as compared to Diamond's model is that this does not contain consumer optimisation, and one added, novel feature is that the expansion of human capital is among the explanatory variables of the model. Its core concept is the trade-off between quantity and quality as formulated by Becker (*Becker 1960, Becker – Lewis 1973, Willis 1973, Barro – Becker 1989, Becker et al. 1990, Galor – Weil 1999*), according to which adults spend more on their offspring when they have less children, i.e. they provide them with more human capital. As a result, when children grow up, their productivity increases, and therefore the impact of the lower total fertility rate can be offset. Our paper shows that the total fertility rate can drop to a critical level where the opportunity for growth is threatened, and society is unable to maintain the earlier level of GDP per capita. The study is structured as follows: after the introduction, in the second part, we describe Becker's quantity–quality trade-off, and the corresponding theories. In Part 3, we will present our own model, and show in which aspects the framework developed by us differs, and to what extent, from *Lee – Mason* (*2010*). In Part 4, we use artificially constructed exogenous fertility and mortality rates, and simulate paths for GDP per capita based on the model of the previous part. We detail the demographic trends that the exogenous total fertility and mortality rates express, and why it is important to be aware of the impact these trends may have on economic growth. Part 5 comprises the summary of our study. Then in the Appendix we provide an overview of the historical development of growth models using demographics in a tabular form, and we also employ a table to show the values of the parameters in our model.

2. Beckerian quantity–quality trade-off

The basic idea behind the Beckerian quantity–quality trade-off is attributed to *Becker* (*1960*) in the literature. In short, the quantity–quality trade-off means that if there are many offspring in a family, less time and money are devoted to one, while in the case of fewer offspring, the expenses per child increase drastically. Similar approaches to the quantity–quality trade-off are discussed in *Becker – Lewis* (*1973*), *Willis* (*1973*) and *Galor – Weil* (*1999*). This issue, especially the global contraction of the total fertility rate, is very topical now. For example, *Lee – Mason* (*2010*) use statistical analysis to prove that in the countries in the NTA[5] database, children with fewer siblings receive more human capital investments from their parents; therefore, they can work more efficiently as adults than their peers growing up in larger families. The link between the reduction in the total fertility rate and the increase in human capital investments may explain why production values increase

[5] NTA: National Transfer Accounts. The data can be found at http://www.ntaccounts.org/ (last accessed on 20 September 2017).

even if the share of the working-age population diminishes. *Roudi-Fahimi – Kent* (*2007*) provide an outstanding summary of the studies in connection with this idea.

Even the so-called synthesis theory, which treats the causal relationships in a complex manner, and which does not support the concept of the Beckerian quantity–quality trade-off (e.g. *Adelman 1963*, *Freedman 1963, Silver 1965, Freedman – Coombs 1966a, Freedman – Coombs 1966b and Easterlin 1973*) concedes that the decrease in the total fertility rate and the growing human capital investments occurred at roughly the same time in history. Further schools of thought other than the synthesis theory that distance themselves from Becker's ideas explain the reduction in the fertility rate with several factors. These include the increase in female employment, economic crises, people's love of comfort and the opinion of contemporaries (see, for example, *Kaplan 1994, Black et al. 2005, Ellis 2008, Luci – Thevenon 2010, Sobotka et al. 2011*, *Colleran et al. 2015, Dang – Rogers 2016*). And despite the apparent facts, the hotly debated *Lawson – Borgerhoff Mulder* (*2016*) study flatly rejects that there is any causality between human capital investments and the development of the total fertility rate. Nonetheless, these two authors admit that fertility has declined and the human capital invested in children has increased since the mid-19th century. *Guo – Zhang* (*2017*), partly in response to the paper by *Lawson – Borgerhoff Mulder* (*2016*), show that the reason why certain authors doubt the theory of the quantity–quality trade-off is a misguided interpretation of the facts.

One exception to Becker's ideas could be that in the countries with an advanced public sector, all children have access to basic schooling and healthcare, regardless of the number of siblings. Yet *Vargha – Donehower* (*2016*) estimated the value of the so-called invisible transfers,[6] i.e. the care and attention directly devoted to children by their parents. The two authors have confirmed that the children growing up among fewer siblings receive much more care in the countries with advanced public benefits than their peers living in large families, mainly because of the invisible transfers.

In our article, we base our model on the negative relationship between the number of children and human capital investments, i.e. we accept the causality derived from Becker's ideas. We examine what would happen if this correlation determined the productivity of an economy, *ceteris paribus*. The impact of material capital and all the other factors is quantified only indirectly, through human capital, but it is indicated that sometimes material capital can be the driver of development. The approach examining the role of human capital *ceteris paribus* is increasingly common in the literature, in view of the drying-up of the first demographic dividend

[6] For more on the invisible transfers themselves, see *Gál et al.* (*2016*).

and the increasingly dominant role of the second demographic dividend[7] (e.g. *Bloom et al. 2003, Mason 2005, Mason et al. 2016*).

3. Our model

3.1. Differences between *Lee – Mason* (*2010*) and our model

Just as in the case of *Lee – Mason* (*2010*), our overlapping generations model is highly stylised. Our basic aim is to determine the impact of the population's age distribution (and indirectly population numbers) on output per capita. Despite its stylised nature, it is important that the model be as realistic as possible.

From this perspective, taking into account four overlapping generations instead of the three, as in *Lee – Mason* (*2010*), provides enormous help in achieving better alignment with the facts. With three overlapping generations, those living until the end of the third period spend merely one-third of their life working, and live from the transfers provided by the others for two-thirds of their life. In the case of three generations, there is an unreasonably high number of children in any period in the context of the actual TFR. If we assume that $N_t^{working}$ is the number of workers and F_t is the fertility rate (for one person rather than one woman, which is usually the case in such models), then $F_t N_t^{working}$ children will be born altogether, and the share of the first generation will be disproportionately large, even when F_t is barely over 1 (and the TFR is over 2). In the case of four generations, the people living until the end of the fourth period spend half of their lives working, and if the total number of workers in the two working generations is $N_t^{working}$, while the total fertility rate is F_t, then, assuming that the two working generations are roughly of the same size,[8] only $F_t 0.5 N_t^{working}$ children will be born. The share of children within society is much more realistic this way, just like the fact that people spend at least half, rather than a third, of their lives on the labour market. Those who die after the third period, work for two-thirds of their lives.

Accordingly, our model assumes two, rather than one, working generations living at the same time: they are the second and third generations in the model. These two generations resemble each other, but only the second generation can produce children. In addition, the income of the third generation is somewhat higher than that of the second. Our model does not differentiate between whether the transfers or the in-kind goods are provided to children (Generation 1) and the old (or pensioners, i.e. Generation 4 in our model) within the family or

[7] For a short description of the first demographic dividend, see *Footnote 4*. The second demographic dividend refers to the economic growth fuelled by the stronger human capital investments and the related physical capital investments.

[8] The roughly same size of the two working generations is merely a stylised and momentary simplification, with the sole of aim of highlighting the magnitude.

through redistribution. The overall benefits cover the consumption of children and pensioners, and guarantee the human capital investments in children. These benefits together are referred to as transfers.

Somewhat simplifying the complex correlations in reality, we assume in our model that both working generations divide their income based on the same principles between children's transfer, their own consumption and the elderly's consumption. Therefore, both working generations take part in caring for children,[9] and the elderly have no savings at all, i.e. their livelihood depends completely on the transfers provided by the second and third generations.

The generations that do not work, i.e. children and the elderly, may receive transfers from the workers, with variable lower and upper limits, depending on certain factors in our model. Thanks to the limits, those who do not work receive at least minimal benefits, even in hard times. Furthermore, the limits also ensure that the workers do not have to spend on those who do not work beyond their means, even if the number of those who do not work is relatively high. The rules in the model that determine the way the income of the two working generations is spent also ensure that the simulation results are determined not only by the proportion of the size of the individual generations in the population, but also indirectly by the total population. Meanwhile, the amount spent on human capital investments from the transfers influences the productivity of the young workers in the next period, and of the older workers in the era coming after that.

Although in their model Lee and Mason quantify the fertility elasticity of human capital, as the embodiment of Beckerian quantity–quality trade-off, using actual statistical data and an econometric equation, the value of elasticity is a constant negative number. We did not regard the value of elasticity to be constant, but to be dependent on the total fertility rate. The details of the calculations are presented in *Chapter 3.3*.

3.2. Equations in the model

As described in the previous part, the economy in the model contains four overlapping generations:[10] children (N_t^1), young workers (N_t^2), older workers (N_t^3) and pensioners (N_t^4). On average, the members of the young working generation produce F_t children in Period t. In the next period, children become economically active as young workers, and young workers become older workers. Finally, the latter's s_t share will reach retired age, and the others die at the end of the third

[9] According to the model, both working generations finance children's consumption and human capital, but they do not necessarily provide the transfers directly. The distribution in the model is similar to a system in which children receive the amount through central redistribution.

[10] The population number of the generation where i = 1, 2, 3, 4 in Period t is denoted with N_t^i.

period. The demographic transitions in the model are described in the following equations:

$$N_t^1 = F_t \cdot N_t^2 \tag{1a}$$

$$N_t^2 = N_{t-1}^1 \tag{1b}$$

$$N_t^3 = N_{t-1}^2 \tag{1c}$$

$$N_t^4 = s_t \cdot N_{t-1}^3 \ . \tag{1d}$$

Total population size in Period *t*.: $N_t = N_t^1 + N_t^2 + N_t^3 + N_t^4$.

As indicated earlier, in our model F_t denotes the fertility rate for one person rather than one woman in Period *t*. Therefore, assuming an equal number of men and women in a stylised approach, our simulation used half of the TFR value actually possible. In our model, no one dies until the end of the third period, and those surviving the third period die only at the end of the fourth period, but then they do so for sure. Therefore, in our model, s_t is half of the statistically measurable survival rate in the third period. The development of F_t and s_t is also key in the model. The two exogenous ratios together determine the size and structure of the population in each period.

Only two working generations perform income-generating activities, and they receive wages for their work. The wage of young workers (W_t^2) depends on the amount of human capital they possess (H_t), which was accumulated in the previous period:

$$W_t^2 = g\left(H_t\right), \tag{2a}$$

where $g'(H_t) > 0$ and $g''(H_t) < 0$. The wages of older workers are proportionately higher than that of younger ones, which is indicated by the parameter φ, the value of which used in our calculations can be seen in *Table A2* of the Appendix.

$$W_t^3 = f\left(W_t^2\right) = \varphi \cdot W_t^2 , \tag{2b}$$

where $\varphi > 1$.

Human capital is invested in children by the two working generations, spending h_t of their income for this purpose. Equation (3) shows the amount of human capital of the young workers entering working age in Period *t*. In line with the assumptions in *Lee – Mason* (*2010*), Equation (3) presumes that everyone receives the human capital investments in childhood, and that this will determine their productivity when they become workers:

$$H_t = h(F_{t-1}) \cdot (W_{t-1}^2 + W_{t-1}^3) \,. \tag{3}$$

Since only the two working generations have income, they finance their own consumption as well as that of the two dependant generations, and the human capital investments in children. Accordingly, the budget constraint is as follows:

$$W_t^2 \cdot N_t^2 + W_t^3 \cdot N_t^3 \geq C_t^1 \cdot N_t^1 + C_t^2 \cdot N_t^2 + C_t^3 \cdot N_t^3 + C_t^4 \cdot N_t^4 + H_{t+1} \cdot N_t^1 \,. \tag{4}$$

In line with the above, the amount of human capital investments financed from income depends on the total fertility rate, based on the following correlation:

$$H_t = \alpha \cdot F_{t-1}^{\beta_{t-1}} \cdot (W_{t-1}^2 + W_{t-1}^3) \,, \tag{5}$$

where $\alpha > 0$ is the human capital investment ratio with one unit of F (replacement level). β_{t-1} is the elasticity of human capital investments as a function of F_{t-1}, which will be later referred to as fertility elasticity. This was fixed by *Lee – Mason* (*2010*) at an average value, but we made it dependent on the total fertility rate. The estimation of β_{t-1} is presented in *Chapter 3.3*.

The wage of the two working generations is

$$W_t^2 = \gamma \cdot \mathrm{H}_t^\delta = \gamma \cdot \left(\alpha \cdot F_{t-1}^{\beta_{t-1}} \cdot (W_{t-1}^2 + W_{t-1}^3)\right)^\delta \,, \tag{6a}$$

where $0 < \delta < 1$, $\gamma > 0$ and substituting Equation (6a) in Equation (2b):

$$W_t^3 = \varphi \cdot \gamma \cdot \left(\alpha \cdot F_{t-1}^{\beta_{t-1}} \cdot (W_{t-1}^2 + W_{t-1}^3)\right)^\delta \,. \tag{6b}$$

We would like to point out parameter γ in Equation (6b), the value of which was fixed at 1 in our model calculations, similar to *Lee – Mason* (*2010*) (see *Table A2* in the Appendix). If γ is greater than 1, this represents a sort of growth factor in the model. This growth factor can express the expansion in the efficiency of either human capital or physical capital. Since we currently aim to monitor the potential changes in fertility and mortality rates *ceteris paribus*, we utilised only one direct efficiency-increasing option in the model, the impact of human capital investments on the efficiency in production. Nonetheless, we allow for the option of the parameter γ being greater than 1 when our simulation is rerun. In this manner, the growth in the efficiency of physical capital can be modelled discretely.

Correlation (4) regarding the budget shows that the different generations can spend more money only at each other's expense. Therefore, we had to incorporate limits in our model that prevent the potentially large first or fourth generations from burning through the income generated by the workers, leaving nothing to those who produced it. In addition, a similar limit had to be applied to prevent the opposite, i.e. we had to ensure that the two working generations provide at least minimal transfers to the first and fourth generations, even if there are many people in the latter groups in a given period. Equations (7a) and (8a) below place caps and floors on the consumption of the elderly and children, respectively, using Ψ_t defined in Equation (7b), μ_t determined in Equation (8b) and ν_t established in Equation (8c).[11] The consumption of the fourth generation is:

$$C_t^4 = \Psi_t \cdot \left(W_t^2 \cdot \frac{N_t^2}{N_t^4} + W_t^3 \cdot \frac{N_t^3}{N_t^4} - H_{t+1} \cdot \frac{N_t^1}{N_t^4} \right), \tag{7a}$$

where

$$\Psi_t = \min\left(0.25;\ \frac{N_t^4}{N_t};\ 1.1 \cdot \frac{N_{t-1}^4}{N_{t-1}} \right). \tag{7b}$$

The consumption of the first generation is:

$$C_t^1 = \mu_t \left(W_t^2 \cdot \frac{N_t^2}{N_t^1} - \alpha \cdot F_t^{\beta_t} \cdot W_t^2 \right) + \nu_t \left(W_t^3 \cdot \frac{N_t^3}{N_t^1} - \alpha \cdot F_t^{\beta_t} \cdot W_t^3 \right), \tag{8a}$$

where

$$\mu_t = \min\left(0.25;\ \frac{N_t^1}{N_t};\ 1.1 \cdot \frac{N_{t-1}^1}{N_{t-1}} \right) \tag{8b}$$

$$\nu_t = \min\left(0.25;\ \frac{N_t^1}{N_t};\ 1.1 \cdot \frac{N_{t-1}^1}{N_{t-1}} \right). \tag{8c}$$

The income left over after the payment of the transfers to the first and fourth generations is spent by younger and older workers on their own consumption:

$$C_t^2 = \left(1 - \mu_t - \Psi_t\right)\left(W_t^2 - \alpha \cdot F_t^{\beta_t} \cdot W_t^2 \cdot \frac{N_t^1}{N_t^2} \right) \tag{9a}$$

$$C_t^3 = \left(1 - \nu_t - \Psi_t\right)\left(W_t^3 - \alpha \cdot F_t^{\beta_t} \cdot W_t^3 \cdot \frac{N_t^1}{N_t^3} \right). \tag{9b}$$

The formula of GDP per capita, which in our model means income per person, should be expressed separately. This can be seen in Equation (10). This is because our simulation paths determine the development of GDP per capita.

$$\frac{GDP_t}{N_t} = \frac{W_t^2 \cdot N_t^2 + W_t^3 \cdot N_t^3}{N_t}. \tag{10}$$

[11] The above-mentioned payment limits were incorporated in the model using the ratios Ψ_t, μ_t and ν_t. If the four generations are of the same size, and the available income (remaining after the payment of human capital) is distributed evenly among them, all generations receive 25 per cent of the income. Otherwise, in the case of generations of a relatively smaller size, the proportion of spending on their consumption changes in line with their share within the population, but it may rise by 10 per cent at most in a period to avoid extreme growth.

Using the previous equations of the model, wage dynamics, i.e. the ratio of future and current wages, can be described as follows:

$$\frac{W_{t+1}^2}{W_t^2}=\frac{\gamma\cdot\left(\alpha\cdot F_t^{\beta_t}\cdot\left(W_t^2+W_t^3\right)\right)^{\delta}}{W_t^2}=\gamma\cdot\left(\alpha\cdot F_t^{\beta_t}\right)^{\delta}\cdot\frac{\left(W_t^2+\varphi\cdot W_t^2\right)^{\delta}}{W_t^2}=\gamma\cdot\left(\alpha\cdot F_t^{\beta_t}\right)^{\delta}\cdot\left(1+\varphi\right)^{\delta}\cdot W_t^{2^{\delta-1}} \quad (11a)$$

$$\frac{W_{t+1}^3}{W_t^3}=\frac{\varphi\cdot W_{t+1}^2}{\varphi\cdot W_t^2}\,. \quad (11b)$$

In a steady state, wages are constant, and the wage of young workers is

$$W_t^{2^*}=\left(\frac{1}{\gamma\cdot\alpha^{\delta}}\right)^{\frac{1}{\delta-1}}\cdot F_t^{\frac{\beta_t\delta}{1-\delta}}\cdot\left(\frac{1}{\left(1+\varphi\right)^{\delta}}\right)^{\frac{1}{\delta-1}}, \quad (12a)$$

while, when substituting Equation (12a) in (2b), the wage of older workers is

$$W_t^{3^*}=\varphi\cdot\left(\frac{1}{\gamma\cdot\alpha^{\delta}}\right)^{\frac{1}{\delta-1}}\cdot F_t^{\frac{\beta_t\delta}{1-\delta}}\cdot\left(\frac{1}{\left(1+\varphi\right)^{\delta}}\right)^{\frac{1}{\delta-1}}. \quad (12b)$$

3.3. Estimating the fertility elasticity of human capital investments

In *Chapter 3.1*, we already indicated that *Lee – Mason* (*2010*) determined the value of the elasticity of human capital investments as a function of the total fertility rate to be negative but constant. The two authors used *NTA* (*2009*) data and found a significant negative correlation between the total fertility rate and human capital investments. However, they relied on this relationship as a generally occurring link, quantifying the elasticity of human capital investments as a function of the total fertility rate based on the average value.

By contrast, we employed elasticity that depends on the value of the total fertility rate. We used the average number of years spent in school (ISCED 1 or higher level studies) as a variable accurately representing the amount of human capital investments from the *UNESCO* (*2016*) database. The value of the total fertility rate was compiled from *UN* (*2015*), and in the end we had data for 98 countries.[12] The countries were divided into two groups, one for the countries with a TFR of over 2.1 (the replacement level), and the other for the countries below that. Using the ordinary least squares (OLS) method, we prepared regression estimates for the two country groups separately. In the case of high-fertility countries, the absolute value of the regression coefficient was significantly higher: β_t = –0.8348, while it

[12] We were able to use data from far more countries than *Lee – Mason* (*2010*), who relied solely on *NTA* (*2009*).

was β_t = –0.273 in the low-fertility country group. After that, assuming a linear relationship[13] between β_t and the F_t fertility rate, β_t was determined as follows:

$$\beta_t = -\,0.4072 - 0.761 \cdot \mathit{In}F_t\,. \tag{13}$$

The values determined by Equation (13) fit well on the points represented by the total fertility rate and human capital investments of the 98 countries, and that is why this correlation was chosen in our model for estimating the elasticity of human capital investments as a function of the total fertility rate. If $F_t < 0.588$, β_t would be positive, therefore in this range, Equation (13) conflicts with the theory based on which we quantified it. However, there was no such low total fertility rate in the statistical data, and we did not use such low F_t values in our simulation. Further research is necessary to confirm or reject the relevance of the Beckerian quantity–quality trade-off with respect to TFR values below 1.176. Yet during our model calculations, we never used such a low total fertility rate.

4. The relationship between the total fertility rate, the survival rate and growth based on our model's correlations

The simple structure of our model facilitates simulation testing that examines the impact of changes in only the total fertility rate and the survival rate on economic growth, independent of other factors. These simulation calculations show the impact of the two aforementioned demographic indicators on the development of GDP per capita, all other factors being fixed. In other words, only the categories determining the size of the population[14] change in our model, while everything else is constant.

Assuming simple correlations has both benefits and drawbacks. The most important benefits in this case are:

- We can see the effects exerted by the simultaneous change in the total fertility rate and the mortality rate *ceteris paribus*.
- We can analyse how the different paths of the total fertility rate and the mortality rate as compared to each other change per capita output.
- We can highlight the cases when the expected demographic developments lead to a dangerous economic situation.

[13] It was assumed that the points determined by the average total fertility rate and human capital investments (or, more precisely, its logarithm), can be connected with a straight line. The mathematical statement of this line can be seen in Equation (13).

[14] There is no migration in our model.

- If demographic developments influence economic growth favourably, even temporarily, the projection helps the economy in utilising the positive trends as long as possible.

The drawbacks of the simple approach are the following:

- Several factors can influence GDP per capita that cannot be derived from the development of the total fertility rate and the survival rate.
- Our forecasts can stoke unreasonable fears if the negative impact of demographic factors can be offset.
- Positive predictions can falsely suggest that nothing has to be done for economic growth, as the demographic drivers take care of that “on their own”.

Now we will present the GDP per capita values on the various simulated paths. During our simulations, both the total fertility rate (F) and the survival rate (s) were provided exogenously.[15]

The exogenous time series for F and s were determined in a way so that there would be cases when a high fertility rate nosedives, when the moderately high fertility rate stay roughly the same, and when the low fertility rate continues to decline. In parallel with this, the survival rate stagnates or grows in all cases. Therefore, we sought to find out how the various possible paths of the total fertility rate influence GDP per capita *ceteris paribus*, while survival rates do not drop. Of course, our model can be used to conduct simulations using several other fertility and mortality time series, although we mainly focused on the various, hypothetical and opposite paths of the total fertility rate, while assuming that the mortality rate values do not deteriorate at all.

We started our simulation from a steady state, from which the economy shifts due to the change in the total fertility rate and the survival rate. The values of the parameters used during the simulation can be seen in *Table A2* of the Appendix. Similar to *Lee – Mason* (*2010*), the initial wage in the steady state is the wage value in the context of the given parameters and exogenous variables. After the total fertility rate and the survival rate start changing in each period, the economy shifts from this steady state, and later, when F and s do not change anymore, it converges towards a new steady state. Similar to *Lee – Mason* (*2010*), our study focused on the path towards the new steady state, i.e. the time of transition.

[15] As a reminder: F is the fertility rate per person, and half of the expected TFR, while s is the survival rate, and half of the survival rate for 60 years of age supported statistically, since in the model those living to be 60 die at the age of 80.

Several other authors have used a modelling technique in which the stationary condition of the economy was upset, and it was examined how the path leading to the new steady state developed. For example, *Cipriani* (*2014*) used a steady state of an OLG model to demonstrate the expected impact of rising life expectancy in the context of first an exogenous, then an endogenous total fertility rate. He wanted to find out how much pension will be paid to the members of the older generation. *Becker et al.* (*1990*) also examined the steady state and its stability using endogenous fertility and human capital. The model by *Barro – Becker* (*1989*) administered hypothetical shocks to the steady state, and monitored the values of the variables in the model. *Kalemli-Ozcan et al.* (*2000*) analysed the impact of the change in the mortality rate in the context of variable and constant educational attainment, and this model also started out from a steady state. To mention some Hungarian examples, *Simonovits* (*2009*) and *Varga* (*2014*) upset the steady state of their models, which were very similar to each other in many respects, and examined how the parametric pension reforms affected the sustainability of the pension system.

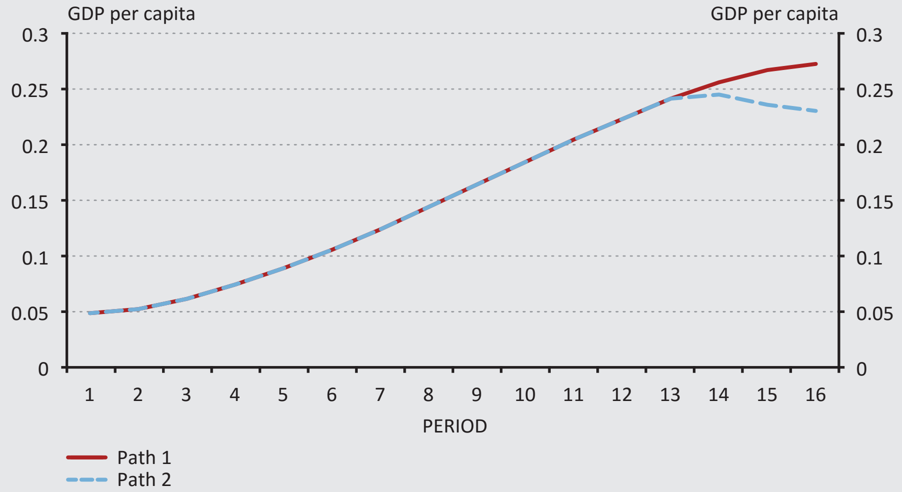
We also assumed in our model that the economy was in a stationary state in the 0th period before modelling. Then the total fertility rates and mortality rates changed from period to period. We sought to find out how the changes shape GDP per capita. *Table 1* contains the values of *s* and *F* used in the first and second simulation calculations in the 16 periods under review, while *Figure 1* presents the development of GDP per capita corresponding to these (*s,F*) pairs.

The growth in GDP per capita along the first path clearly shows that if the initially very high total fertility rate continuously declines, even sinking well below the replacement level by the end of the period under review, GDP per capita may increase even in the context of a high survival rate. The survival rate increases steadily and at a relatively fast pace for 13 periods along the first path, and then during the final three periods, it stabilises at the high level achieved until then. GDP per capita grows even in these final three periods, although more moderately than before.

Table 1
Total fertility and survival rates along the first and second simulation paths

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Path 1	F	3.50	3.15	2.84	2.56	2.31	2.08	1.87	1.69	1.52	1.37	1.23	1.11	1.00	0.90	0.81	0.73
	s	0.20	0.22	0.23	0.25	0.27	0.29	0.32	0.34	0.37	0.40	0.43	0.47	0.48	0.48	0.48	0.48
Path 2	F	3.50	3.15	2.84	2.56	2.31	2.08	1.87	1.69	1.52	1.37	1.23	1.11	1.00	1.05	1.10	1.16
	s	0.20	0.22	0.23	0.25	0.27	0.29	0.32	0.34	0.37	0.40	0.43	0.47	0.48	0.48	0.48	0.48

Figure 1
GDP per capita values for Growth Paths 1 and 2 according to Table 1



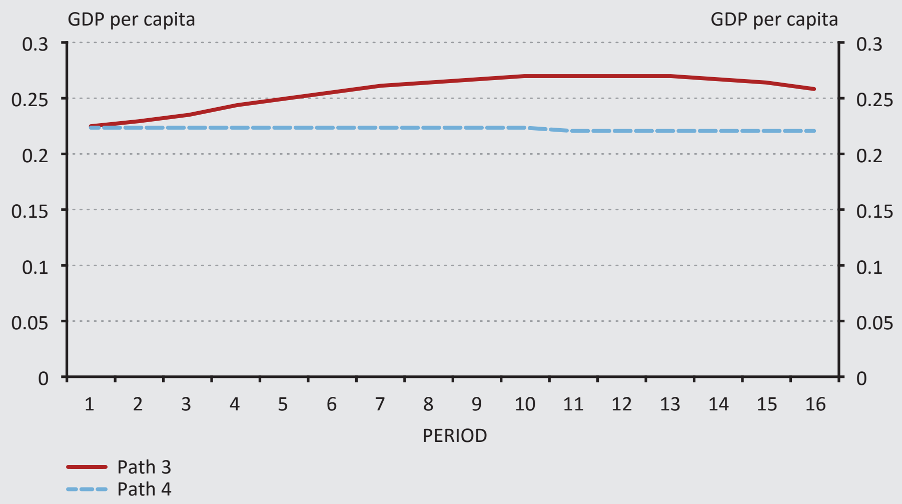
The simulation was performed on Path 2 using the same survival rates as in Path 1, and the total fertility rates were also almost the same (*Figure 1*). The total fertility rate starts out from the same high level in the first period and then falls to the replacement level, just like along Path 1. However, upon reaching the replacement level, it starts increasing slightly instead of continuing on its downward path. In parallel with the hike in the total fertility rate, GDP per capita drops instead of growing as before. Therefore, our model suggests that if we use only the total fertility rate and the mortality (survival) rates as exogenous variables, the continuous contraction of the total fertility rate to any low level is favourable from an economic growth perspective under all conditions. Yet the simulation results along Paths 3 and 4 (*Table 2*) highlight the fact that the drop in the total fertility rate can reach a drastic level in our model where it entails a decrease in GDP per capita.

Table 2
Total fertility and survival rates along the third and fourth simulation paths

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Path 3	F	1.20	1.14	1.09	1.04	0.99	0.94	0.90	0.85	0.81	0.77	0.74	0.70	0.67	0.64	0.61	0.59
	s	0.40	0.41	0.41	0.42	0.42	0.43	0.44	0.44	0.45	0.45	0.46	0.47	0.47	0.48	0.49	0.49
Path 4	F	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20	1.20
	s	0.40	0.41	0.41	0.42	0.42	0.43	0.44	0.44	0.45	0.45	0.46	0.47	0.47	0.48	0.49	0.49

If the already high survival rate steadily increases, in an extreme case assuming for the last period that almost all 60-year-olds will live until retirement age, and thus also live to be 80 in our model, GDP per capita falls in the context of an excessively contracting total fertility rate. Using the same hypothetical survival rate for our simulation calculations, but employing constant total fertility rates slightly above the replacement level, GDP per capita exhibits only a minimal decline. Nevertheless it seems that the development of GDP per capita is most affected in our model by the growth or decrease in the total fertility rate. This is also suggested by the simulation along Paths 5 and 6 (*Figure 3*).

Figure 2
GDP per capita values for Growth Paths 3 and 4 according to Table 2

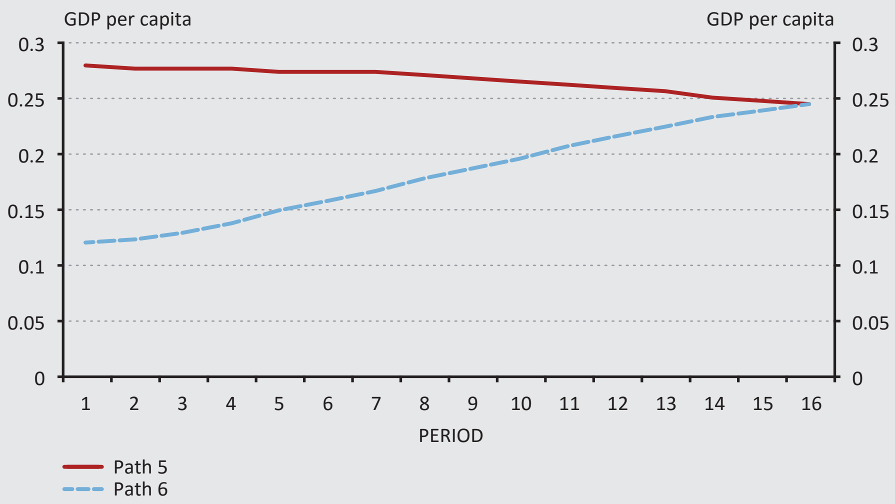


The exogenous total fertility rates and survival rates of Paths 5 and 6 were provided based on a special consideration. The survival rates are the same in both versions, starting out from a moderately high level, and growing relatively slowly but steadily, and only the total fertility rates are different. Along Path 5, the total fertility rates that start out from below the replacement level increase continuously, right until the last period, where they are slightly over the replacement level. However, along Path 6, the total fertility rates that start out from a level more than twice the replacement level drop continuously. As a result of the above, the GDP per capita values along Paths 5 and 6 are quite different from each other. They continuously decline along Path 5, while they steadily grow along Path 6. The initial value is very low on Path 6, but after a consistent rise, it is only infinitesimally lower in the last period than the corresponding value on Path 5. The levels achieved on the paths should be compared to the values along the same path – one only need to think of the hurdles described at the beginning of the chapter – however, the contrasting development of the GDP values in the first and last periods clearly shows the opposing dynamics of the two paths.

Table 3
Total fertility and survival rates along the fifth and sixth simulation paths

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Path 5	F	0.65	0.67	0.69	0.72	0.74	0.77	0.79	0.82	0.85	0.88	0.91	0.94	0.97	1.00	1.04	1.05
	s	0.35	0.36	0.36	0.37	0.37	0.38	0.38	0.39	0.39	0.40	0.41	0.41	0.42	0.42	0.43	0.44
Path 6	F	2.10	2.00	1.90	1.81	1.73	1.65	1.57	1.49	1.42	1.35	1.29	1.23	1.17	1.11	1.06	1.05
	s	0.35	0.36	0.36	0.37	0.37	0.38	0.38	0.39	0.39	0.40	0.41	0.41	0.42	0.42	0.43	0.44

Figure 3
GDP per capita values for Growth Paths 5 and 6 according to Table 3



5. Summary

Our study summarised how, after the initially simple approaches, the various types of models analysing economic growth took into account the changes in the size and composition of the population. From the perspective of our present article, the simple OLG model presented in *Lee – Mason* (*2010*) is key. We performed our simulation calculations with a model that can be considered an enhanced version of *Lee – Mason* (*2010*).

Our simulation results clearly prove that we can draw many valuable conclusions even if, similar to the model by Lee and Mason, we assume that the only factors that determine the path of the OLG model are the total fertility rate and the survival rate. In such cases, the implicit level of physical capital is fixed, and the increasing efficiency of physical capital materialises in the expansion of the efficiency of human capital investments. Therefore, output per capita in itself does not provide meaningful information, however, the comparison of the levels by periods taught us many lessons.

The general trends observed in the model included the following:

- The change in fertility influenced GDP per capita much more than the shifts in the survival rate.
- The survival rate had a substantial impact when the total fertility rate was well below the replacement level. When the total fertility rate per person (for both men and women) was slightly below 0.6 after declining steadily from period to period, while the survival rate continuously increased, GDP per capita started to dip. In such cases, the survival rate, which increased and approximated 1 (meaning an s value converging towards 0.5 in our model at 60–80 years of age), prevented the growth of GDP per capita.
- When comparing two simulation calculations in which the survival rates were the same in each period, we mostly saw that the increasing and more steeply increasing GDP per capita values were on the path along which the total fertility rates declined or declined more, even to below the replacement level. However, it was important not to let the total fertility rates fall as low as we described in the previous paragraph.
- When on one of two model paths containing the same survival rates the total fertility rate increased from period to period from a low initial value, and it declined from an initial high level on the other path, the first path indicated a steadily decreasing output per capita, while the other exhibited a consistently expanding value, even if the total fertility rates were the same in the last period. These two simulations showed the most clearly that such models “reward” the reduction in population and “punish” population growth.

No hasty conclusions should be drawn from the above. For example we cannot say that a drop in fertility benefits economic growth, since the combined effect of several other factors also needs to be taken into account in connection with this issue. We should also not forget that if the Beckerian quantity–quality trade-off does not take hold, for example the parents do not provide substantial human capital investments to a low number of children, all the conclusions of our model instantly become unrealistic. Nonetheless, based on the simulation calculations of our model, in most cases it can be assumed that the gradual decrease of the total fertility rate to slightly below the replacement level has a beneficial effect on the development of GDP per capita.

Appendix

A.1. Demographics in growth models

Since ours is a demographic growth model, the role of demographic variables in the development of growth models should be summarised. This can be seen *Table A1*.

Table A1
The role of population in the various growth models

Type of model	Creators of the model	The way population is taken into account	Size of the population, and its role in growth
Classical	Thomas Robert Malthus	Population grows more rapidly than the amount of food.	Excessive population growth leads to lower prosperity.
Keynesian	Roy F. Harrod, Evsey Domar	Exogenous savings ratio, no consumer optimisation.	Economic growth does not depend on it.
Neoclassical	Robert M. Solow, Trevor W. Swan	There is no consumer optimisation, the labour force and productivity of the population influences economic output.	The change in exogenous population growth influences the development of income per capita only in the period of convergence towards the equilibrium growth path, and does not do so in the long term.
Neoclassical	Frank P. Ramsey, David Cass, Tjalling C. Koopmans	Households decide on their consumption and savings path, maximising their own utility.	Short-term dynamics diverge from the Solow–Swan model, but the increase in income per capita does not depend on exogenous population growth in the long run.
Endogenous growth	Kenneth J. Arrow, Paul M. Romer, Robert E. Lucas, Sergio Rebelo	The development of workers' productivity is endogenous.	Using certain parametrisation, the externality and R&D based models exhibit a positive relationship between the exogenous population growth rate and the increase in GDP per capita.
Overlapping generations	Paul A. Samuelson, Peter A. Diamond Alan J. Auerbach, Laurence J. Kotlikoff	There are several overlapping generations, maximising their lifetime utility.	Population growth is exogenous, but the composition of the population may vary, and there are transactions across generations.
Endogenous fertility	Gary S. Becker, Robert J. Barro	Utility-maximising households also decide about the number of offspring.	Fertility develops in line with the optimal consumer decision.
Growth models based on national transfer accounts	Ronald Lee, Andrew Mason	Consumption and income indicators by age.	The impact of the first and second demographic dividend on growth depends on the composition of the population.

A.2. Parameter values used during the simulation

Table A2
Parameters of the model

Value	Source
$\alpha = 0.075$	Authors' calculation based on the *NTA* (*2016*) database[1]
$\gamma = 1$	*Lee – Mason (2010)*
$\delta = 0.33$	*Mankiw et al. (1992), Lee – Mason (2010)*
$\varphi = 1.128$	Authors' calculation based on the *NTA* (*2016*) database[2]

[1] *The values of all the countries, 19 in all, for which the NTA (2016) contained complete data were taken into account (Austria, Brazil, Costa Rica, Finland, Hungary, India, Indonesia, Japan, Kenya, Mexico, Nigeria, Philippines, Slovenia, South Korea, Spain, Sweden, Taiwan, Thailand, United States). We found that the human capital investment rate (the share of education spending per child within the income of a 21–40-year old and that of a 41–60-year-old) was 0.075 on average.*

[2] *The information from the 19 countries with complete data were taken into account (NTA 2016) here as well. We saw that the earned income of the workers in the age group of 41–60 was higher by 12.8 per cent on average than that of their peers in the age group of 21–40.*

References

Adelman, I. (1963): *An Econometric Analysis of Population Growth*. The American Economic Review, 53(3): 314–339.

Auerbach, A.J. – Kotlikoff, L.J. (1987): *Dynamic fiscal policy*. Cambridge: Cambridge University Press.

Barro, R.J. – Becker, G.S. (1989): *Fertility Choice in a Model of Economic Growth*. Econometrica, 57(2): 481–501. https://doi.org/10.2307/1912563

Becker, G.S. (1960): *An Economic Analysis of Fertility*. In: National Bureau of Economic Research, (Ed): Demographic and Economic Change in Developed Countries, pp. 209–240.

Becker, G.S. – Lewis, H.G. (1973): *On the Interaction between the Quantity and Quality of Children*. Journal of Political Economy, 81(2): S279–S288. https://doi.org/10.1086/260166

Becker, G.S. – Murphy, K.M.– Tamura, R. (1990): *Human Capital, Fertility, and Economic Growth*. Journal of Political Economy, 98(5): S12–37. https://doi.org/10.1086/261723

Black, S.E. – Devereux, P.J. – Salvanes, K.G. (2005): The More the Merrier? The Effect of Family Size and Birth Order on Children's Education. The Quarterly Journal of Economics, 120(2): 669–700. https://doi.org/10.1093/qje/120.2.669

Bloom, D. – Canning, D. – Sevilla, J. (2003): *The demographic dividend: A new perspective on the economic consequences of population change*. Rand Corporation.

Cipriani, G.P. (2014): *Population aging and PAYG pensions in the OLG model*. Journal of Population Economics, 27(1): 251–256. https://doi.org/10.1007/s00148-013-0465-9

Colleran, H. – Jasienska, G. – Nenko, I. – Galbarczyk, A. – Mace, R. (2015): *Fertility decline and the changing dynamics of wealth, status and inequality*. Proceedings of the Royal Society B: Biological Sciences, 282(1806).

Dang, H.-A.H. – Rogers, F.H. (2016): *The Decision to Invest in Child Quality over Quantity: Household Size and Household Investment in Education in Vietnam*. The World Bank Economic Review, 30(1): 104–142.

Diamond, P.A. (1965): *National Debt in a Neoclassical Growth Model*. The American Economic Review, 55(5): 1126–1150.

Easterlin, R.J. (1973): *Does Money Buy Happiness?* The Public Interest, 30: 3–10.

Ellis, J. (2008): *Culture, fertility, and son preference*. PhD thesis. The London School of Economics and Political Science (LSE).

Freedman, D.S. (1963): *The Relation of Economic Status to Fertility*. The American Economic Review, 53(3): 414–426.

Freedman, R. – Coombs, L. (1966a): *Childspacing and Family Economic Position*. American Sociological Review, 31(5): 631–648. https://doi.org/10.2307/2091855

Freedman, R. – Coombs, L. (1966b): *Economic Considerations in Family Growth Decisions*. Population Studies, 20(2): 197–222. https://doi.org/10.2307/2172982

Frejka, T. (2016): *The demographic transition revisited: a cohort perspective*. Max Planck Institute for Demographic Research, Rostock, Germany, working paper WP–2016–012.

Gál, R.I. – Vanhuysse, P. – Vargha, L. (2016): *Pro-elderly welfare states within pro-child societies: Incorporating family cash and time into intergenerational transfers analysis*. Center for Economic Institutions, Working Paper No 2016–6.

Galor, O. – Weil, D.N. (1999): *From Malthusian Stagnation to Modern Growth*. The American Economic Review, 89(2): 150–154. https://doi.org/10.1257/aer.89.2.150

Guo, R. – Yi, J. – Zhang, J. (2017): *Family Size, Birth Order, and Tests of the Quantity–Quality Model*. Journal of Comparative Economics, 45(2): 219–224. https://doi.org/10.1016/j.jce.2016.09.006

Kalemli-Ozcan, S. – Ryder, H.E. – Weil, D.N. (2000): *Mortality decline, human capital investment, and economic growth*. Journal of Development Economics, 62(1): 1–23. https://doi.org/10.1016/S0304-3878(00)00073-0

Kaplan, H. (1994): *Evolutionary and Wealth Flows Theories of Fertility: Empirical Tests and New Models*. Population and Development Review, 20(4): 753–791. https://doi.org/10.2307/2137661

Lawson, D.W. – Borgerhoff Mulder, M. (2016): *The Offspring Quantity–Quality Trade-off and Human Fertility Variation*. Philosophical Transactions of the Royal Society B: Biological Sciences, 371(1692).

Lee, R. – Mason, A. (2010): *Fertility, Human Capital and Economic Growth over the Demographic Transition*. European Journal of Population / Revue Européenne de Démographie, 26(2): 159–182. https://doi.org/10.1007/s10680-009-9186-x

Luci, A. – Thevenon, O. (2010): *Does economic development drive the fertility rebound in OECD countries?* Working paper No. hal-00520948, HAL.

Malthus, T. (1798): *An Essay on the Principle of Population, as it Affects the Future Improvement of Society with Remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers*. J. Johnson in St. Paul's church-yard. London.

Mankiw, N.G. – Romer, D. – Weil, D.N. (1992): *A Contribution to the Empirics of Economic Growth*. The Quarterly Journal of Economics, 107(2): 407–437. https://doi.org/10.2307/2118477

Mason, A. (2005): *Demographic transition and demographic dividends in developed and developing countries*. In United Nations expert group meeting on social and economic implications of changing population age structures, vol. 31. Mexico City.

Mason, A. – Lee, R. – Jiang, J.X. (2016): *Demographic dividends, human capital, and saving*. The Journal of the Economics of Ageing, 7: 106–122. https://doi.org/10.1016/j.jeoa.2016.02.004

NTA (2009): National Transfer Accounts, http://ntaccounts.org. Downloaded: 1 June 2014.

NTA (2016): National Transfer Accounts, http://ntaccounts.org. Downloaded: 10 June 2016.

Roudi-Fahimi, F. – Kent, M.M. (2007): *Challenges and Opportunities – The Population of the Middle East and North Africa*. Population Bulletin, 62(2).

Silver, M. (1965): *Births, Marriages, and Business Cycles in the United States*. Journal of Political Economy, 73(3): 237–255. https://doi.org/10.1086/259013

Simonovits, A. (2009): *Népességöregedés, tb-nyugdíj és megtakarítás – parametrikus nyugdíjreformok (Population aging, the public pension system and savings: Parametric pension reforms)*. Közgazdasági Szemle, 56(4): 297–321.

Smith, A. (1776): *An inquiry into the wealth of nations*. London: Strahan and Cadell.

Sobotka, T. – Skirbekk, V. – Philipov, D. (2011): *Economic Recession and Fertility in the Developed World*. Population and Development Review, 37(2): 267–306. https://doi.org/10.1111/j.1728-4457.2011.00411.x

Solow, R.M. (1956): *A Contribution to the Theory of Economic Growth*. The Quarterly Journal of Economics, 70(1): 65–94. https://doi.org/10.2307/1884513

UNESCO (2016): *Education: Mean Years of Schooling*. http://data.uis.unesco.org/Index.aspx?queryid=242. Accessed: 20 May 2016.

UN (2015): United Nations, Department of Economic and Social Affairs, Population Division: *World Population Prospects: The 2015 Revision*, DVD Edition.

Van de Kaa, D.J. (2010): *Demographic transition*. Encyclopedia of Life Support Systems, 1, pp. 65–103.

Varga, G. (2014). *Demográfiai átmenet, gazdasági növekedés és a nyugdíjrendszer fenntarthatósága* (*Demographic transition, economic growth and sustainability of the pension system*). Közgazdasági Szemle 61(11): 1279–1318.

Vargha, L. – Donehower, G. (2016): *The Quantity-Quality Tradeoff: A Cross-Country Comparison of Market and Nonmarket Investments per Child in Relation to Fertility*. Paper presented at the European Population Conference, 1–3 September 2016, Mainz.

Willis, R.J. (1973): *A New Approach to the Economic Theory of Fertility Behavior*. Journal of Political Economy, 81(2), pp. S14–S64. https://doi.org/10.1086/260152